

पुस्तिका की सील तब तक न खोलें, जब तक आपको ऐसा करने के लिए कहा न जाए
DO NOT OPEN THE SEAL OF THE BOOKLET UNTIL YOU ARE TOLD TO DO SO



EA SSC-JE MAINS-2022

उत्तर-पुस्तिका (ANSWER BOOK)

(जिन अभ्यर्थियों की उत्तर-पुस्तिका पर अनुक्रमांक, विषय, हस्ताक्षर तथा बाएँ हाथ के अंगूठे की छाप, जहाँ आवश्यक हो, नहीं होंगे उनका मूल्यांकन नहीं किया जाएगा तथा ऐसे अभ्यर्थियों को शून्य अंक दिए जाएँगे।) (Answer Books not bearing Roll No., Subject, Signature and Left-hand Thumb Impression, wherever required, will not be evaluated and such candidates shall be awarded zero mark.)	अनुक्रमांक Roll No.	<input type="text"/>
	विषय Subject	-----
	हस्ताक्षर Signature	-----
	बाएँ हाथ के अंगूठे की छाप Left-hand Thumb Impression	<input type="text"/>

भाषा : केवल एक पर निशान लगाएँ
Language : Tick only one

अंग्रेजी/ENGLISH	<input type="checkbox"/>
हिन्दी/HINDI	<input type="checkbox"/>

टिप्पणी : अभ्यर्थी प्रश्न-पत्र-सह-उत्तर-पुस्तिकाओं में उनके द्वारा भरे/दिए गए विवरण के लिए उत्तरदायी हैं।

Note : Candidates are responsible for particulars filled in/affixed by them in the Question Paper-cum-Answer Books.

निरीक्षक का पूरा नाम / Full Name of Invigilator	निरीक्षक के हस्ताक्षर / Signature of Invigilator
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प्रत्येक अभ्यर्थी को एक उत्तर-पुस्तिका मिलेगी, प्रत्येक में आवरण पृष्ठ सहित 36 पृष्ठ होंगे। यह उत्तर-पुस्तिका सिविल अभियांत्रिकी के लिए है। अभ्यर्थी इस उत्तर-पुस्तिका के ऊपरी हिस्से पर अपनी टिकट संख्या, अनुक्रमांक, विषय लिखें तथा हस्ताक्षर एवं बाएँ हाथ के अंगूठे का निशान लगाएँ। जिन अभ्यर्थियों की उत्तर-पुस्तिका के आवरण पृष्ठ पर अनुक्रमांक, विषय, हस्ताक्षर तथा बाएँ हाथ के अंगूठे का निशान नहीं भरे होंगे उन्हें जाँचा नहीं जाएगा व ऐसे अभ्यर्थियों को "शून्य" अंक दिया जाएगा।

अभ्यर्थियों को सख्त रूप से सलाह दी जाती है कि वे अपनी उत्तर-पुस्तिका के अंदर कोई व्यक्तिगत परिचय, उदाहरणार्थ, नाम, अनुक्रमांक, मोबाइल नम्बर, पता, आदि नहीं लिखें। अभ्यर्थियों द्वारा इस तरह का व्यक्तिगत विवरण लिखना गंभीरता से लिया जाएगा और ऐसी उत्तर-पुस्तिकाओं का मूल्यांकन नहीं किया जाएगा। कोई अतिरिक्त/अनुपूरक उत्तर-पुस्तिका नहीं दी जाएगी।

Each candidate will get one Answer Book, containing 36 pages including the cover pages, This Answer Book is for CIVIL ENGINEERING. Candidates are required to write their Ticket No., Roll Number, Subject and affix their signature and Left-hand Thumb Impression on the cover page of the Answer Book. Answer Book not bearing Candidates's Roll No., Subject, Signature and Left-hand Thumb Impression will not be evaluated and such candidates would be awarded "ZERO" mark.

Candidates are strictly advised not to write any personal identity, e.g., Name, Roll No., Mobile No., Address, etc. inside the Answer Book. Writing of such personal details by the candidates will be viewed seriously and such Answer Book SHALL NOT be evaluated.

No extra/supplementary Answer Book will be provided.

तालिका/TABLE

प्रश्न संख्या Question No.	अधिकतम अंक Maximum Marks	प्राप्तांक Marks Secured (परीक्षक द्वारा भरा जायेगा) (To be filled by Examiner)
1.	60	
2.	60	
3.	60	
4.	60	
5.	60	
6.	60	
जोड़ (अंकों में) Total (in figures)		

प्रश्न-पत्र - II (सिविल अभियांत्रिकी)
PAPER - II (CIVIL ENGG.)

SOM + TOS

Total Marks (in words)

Hundred	Ten	Unit

Note : Attempt any 5 (किन्ही 5 प्रश्नों को हल करें।)

परीक्षक के हस्ताक्षर / Signature of Examiner

इस पृष्ठ पर कुछ न लिखें ।
DO NOT WRITE ANYTHING ON THIS PAGE.

रफ कार्य के लिए दाहिनी ओर मार्जिन का प्रयोग करें, यदि आवश्यक हो ।
USE RIGHT HAND SIDE MARGIN FOR ROUGH WORK, IF NECESSARY.

1.(a) Determine the crippling load for a T-section of dimension $10\text{ cm} \times 10\text{ cm} \times 2\text{ cm}$ and of length 5 m when it is used as strut with both of its ends hinged. Take Young's modulus, $E = 2 \times 10^5\text{ N/mm}^2$ (overall depth = 10 cm , width of flange = 10 cm , width of web = 2 cm and thickness of flange = 2 cm).

T-सेक्शन जिसकी आयाम $10\text{ cm} \times 10\text{ cm} \times 2\text{ cm}$ है और लम्बाई 5 m है, के लिए क्रिपलिंग लोड का निर्धारण करें। जिसे दोनों सिरे हिन्ज्ड स्ट्रट के लिए उपयोग किया जाता है। यंग मॉड्यूलस $E = 2 \times 10^5\text{ N/mm}^2$ कुल गहराई = 10 cm , फ्लेन्ज की चौड़ाई = 10 cm , वेब की चौड़ाई = 2 cm और फ्लेन्ज की मोटाई = 2 cm है।

[20 Marks]

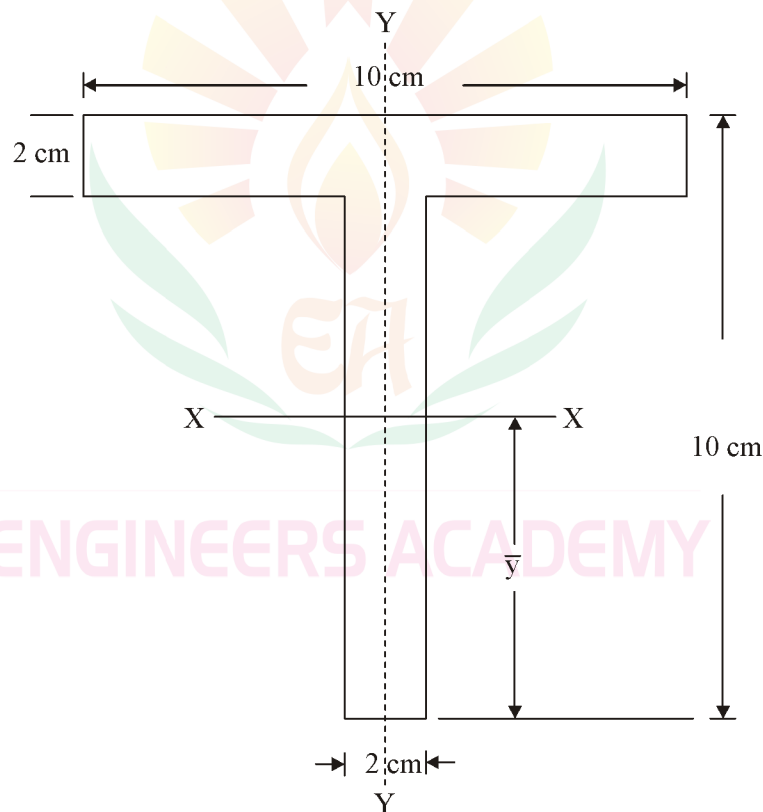
Sol.:

Given : Dimensions of T-section = $10\text{ cm} \times 10\text{ cm} \times 2\text{ cm}$

Length actual, $l = 5\text{ m} = 5000\text{ mm}$

Young's modulus, $E = 2.0 \times 10^5\text{ N/mm}^2$.

First of all, calculate the C.G. of the section. The given section is symmetrical about the axis Y-Y, hence the C.G. of the section will lie on Y-Y axis.



Let

\bar{y} = Distance of C.G. of the section from bottom end.

For the flange, we have

$$a_1 = 10 \times 2 = 20\text{ cm}^2$$

y_1 = Distance of C.G. of area a_1 from the bottom end

$$= 8 + 1 = 9\text{ cm}$$

For the web, we have

$$a_2 = 8 \times 2 = 16\text{ cm}^2$$

y_2 = Distance of C.G. of area a_2 from bottom end

$$= \frac{8}{2} = 4 \text{ cm}$$

Using the relation,

$$\begin{aligned} \bar{y} &= \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} \\ &= \frac{20 \times 9 + 16 \times 4}{20 + 16} = \frac{180 + 64}{36} = 6.777 \text{ cm} \end{aligned}$$

Moment of inertia of the section about the axis X-X,

$$\begin{aligned} I_{XX} &= \left(\frac{10 \times 2^3}{12} + 20 \times 2.223^2 \right) + \left(\frac{2 \times 8^3}{12} + 16 \times 2.777^2 \right) \\ &= (6.667 + 98.834) + (85.333 + 123.387) \\ &= 314.221 \text{ cm}^4 \end{aligned}$$

Moment of inertia of the section about the axis Y-Y,

$$\begin{aligned} I_{YY} &= \frac{2 \times 10^3}{12} + \frac{8 \times 2^3}{12} \\ &= 166.67 + 5.33 = 172 \text{ cm}^4 \end{aligned}$$

Least value of moment of inertia is about Y-Y axis

\therefore

$$I = 172 \text{ cm}^4 = 172 \times 10^4 \text{ mm}^4$$

Since the strut is hinged at both of its end

\therefore Effect length,

$$L_e = l = 5000 \text{ mm}$$

Let

P = Crippling load

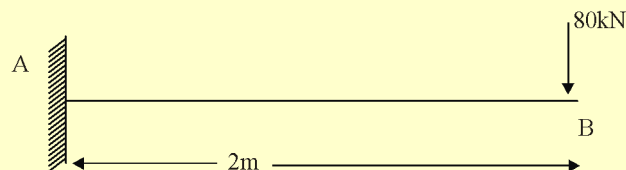
Using equation, we get

$$\begin{aligned} P &= \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 \times 2.0 \times 10^5 \times 172 \times 10^4}{5000^2} \\ &= 135805.7 \text{ N} \end{aligned}$$

Ans.

1.(b) A Cantilever beam 20 cm wide and 25 cm deep projects 2m out of a wall and is carrying a point load of 80 kN at the free end as shown in Fig. Find the slope and deflection of the cantilever at the free end. Take $E = 210 \text{ GPa}$.

एक 2m कैंटीलीवर धरन जिसकी चौड़ाई 20cm और गहराई 25cm है जिसमें मुक्त सिरे पर एक 80kN का बिन्दु भार लगाया गया है। जो चित्र में दिखाया गया है कैंटीलीवर धरन के मुक्त सिरे पर ढाल व विक्षेपण ज्ञात कीजिए। जबकि $E = 210 \text{ GPa}$ है।



[10 Marks]

Sol.:

Given data;

$$B = 20 \text{ cm} = 200 \text{ mm}$$

$$D = 25 \text{ cm} = 250 \text{ mm}$$

$$L = 2\text{m} = 2 \times 10^3 \text{ mm}$$

$$W = P = 80 \text{ kN} = 80 \times 10^3 \text{ N}$$

$$\theta_B = ?, y_B = ?$$

$$E = 210 \text{ GPa} = 210 \times 10^3 \frac{\text{N}}{\text{mm}^2}$$

$$\text{Moment of Inertia } I = \frac{BD^3}{12} = \frac{200 \times 250^3}{12} = 260.42 \times 10^6 \text{ mm}^4$$

$$\theta_B = \frac{WL^2}{2EI}$$

$$= \frac{80 \times 10^3 \text{ N} \times (2 \times 10^3 \text{ mm})^2}{2 \times 210 \times 10^3 \times 260.42 \times 10^6 \text{ mm}^4}$$

$$= \frac{80 \times 10^3 \times 4 \times 10^6 \text{ N} \cdot \text{mm}^2}{2 \times 210 \times 10^3 \times 260.42 \times 10^6 \text{ N} \cdot \text{mm}^2}$$

$$= 2.92 \times 10^{-3} \text{ Radian}$$

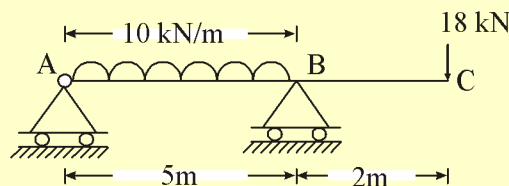
$$y_B = \frac{WL^3}{3EI} = \frac{80 \times 10^3 \times (2 \times 10^3)^3}{3 \times 210 \times 10^3 \times 260.42 \times 10^6}$$

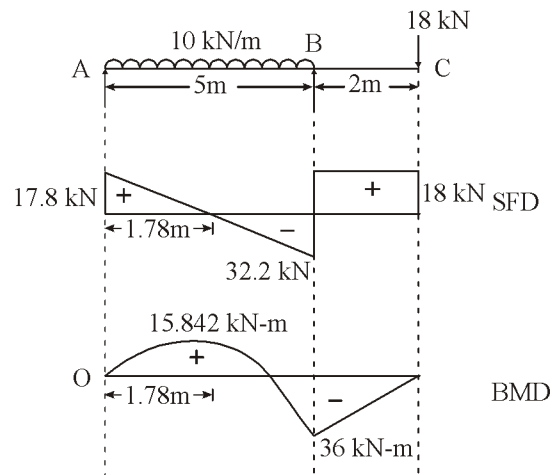
$$= \frac{80 \times 10^3 \times 8 \times 10^9}{3 \times 210 \times 10^3 \times 260.42 \times 10^6} = 3.90 \text{ mm}$$

|| Solution End.

1.(c) Draw the shear force and Bending moment diagrams. Also write down maximum positive and maximum negative value for the both.

नीचे दिये गये चित्र के लिये अपरूपण बल और बंकन आघूर्ण आरेख खींचिए। दोनों के लिए अधिकतम धनात्मक और अधिकतम ऋणात्मक मान भी लिखिए।

**[20 Marks]**

Sol.:

For reactions at A and B.

⇒

$$\Sigma M_A = 0$$

$$5R_B = 251$$

$$R_B = 50.2 \text{ kN}$$

$$R_A + R_B = (10 \times 5) + 18$$

$$R_A = 68 - 50.2$$

$$R_A = 17.8 \text{ kN}$$

Now for shear force

(i) In BC section

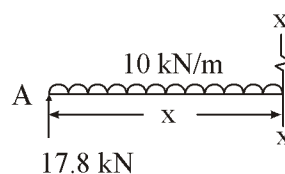


$$SF_C = +18 \text{ kN}$$

$$SF_B = +18 \text{ kN}$$

+ve sign because clockwise direction.

(ii) SF in AB Section



$$SF_A = 17.8 \text{ kN}$$

$$SF_x = 17.8 - 10 \cdot x \text{ (Linear)}$$

at

$$x = 0$$

⇒

$$SF_A = 17.8 \text{ kN}$$

at

$$x = 5$$

⇒

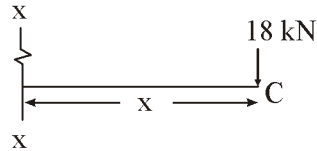
$$SF_B = 17.8 - (10 \times 5)$$

$$SF_B = -32.2 \text{ kN}$$

So, the shear force diagram is shown in fig.

Now for Bending moment :

(i) BM in BC - Section



$$BM_x = -18 \cdot x \text{ (Linear)}$$

Negative sign is due to hogging B.M.

at

$$x = 0;$$

⇒

$$(BM)_C = 0$$

at

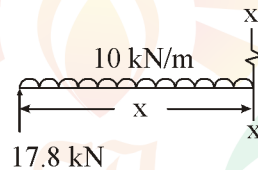
$$x = 2;$$

⇒

$$(BM)_B = -18 \times 2$$

$$(BM)_B = -36 \text{ kN-m}$$

(ii) For AB-Section



$$BM_x = 17.8x - 10 \cdot x \cdot \frac{x}{2} \text{ (2°-parabola)}$$

at

$$x = 0;$$

⇒

$$(BM)_A = 0$$

at

$$x = 5\text{m};$$

⇒

$$(BM)_C = -36 \text{ kN-m}$$

So, BMD is shown by fig.

For maximum BM

$$\frac{dM}{dx} = 0 \Rightarrow 17.8 - 5 \times 2x = 0$$

at

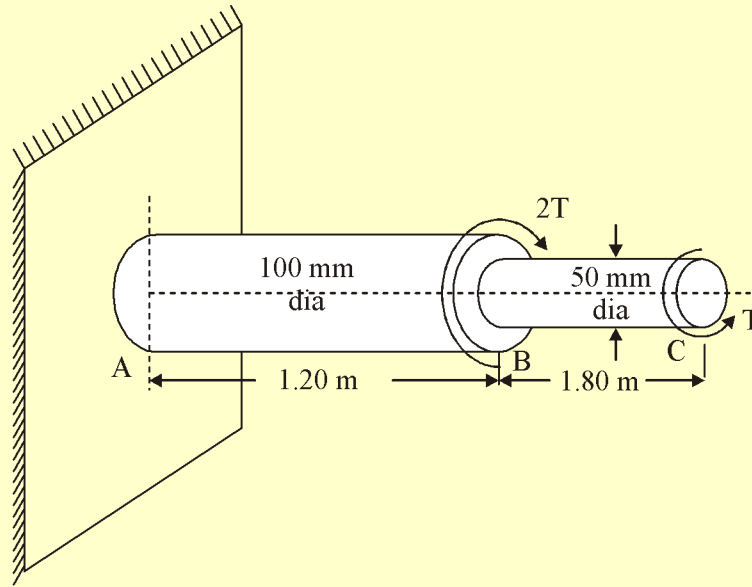
$$x = \frac{17.8}{(5 \times 2)} = 1.78 \text{ m}$$

$$\begin{aligned} (BM)_{\max} &= (17.8 \times 1.78) - (10 \times 1.78 \times \frac{1.78}{2}) \\ &= 15.84 \text{ kN-m} \end{aligned}$$

◀ Solution End.

1.(d) The stepped steel shaft shown in figure is subjected to a torque T at the free end and a torque $2T$ in the opposite direction at the junction of the two sizes. What is the total angle of twist at the free end if the maximum shear stress in the shaft is limited to 70 N/mm^2 . Assume the modulus of rigidity to be $8.4 \times 10^4 \text{ N/mm}^2$.

आकृति में दिखाया गया स्टेप्ड स्टील शाफ्ट के मुक्त सिरे पर T टॉर्क और दोनों भिन्न आकार की बार के जंक्शन पर विपरीत दिशा में $2T$ टॉर्क लगा हुआ है। यदि शाफ्ट में अधिकतम अपरूपण प्रतिबल 70 N/mm^2 तक सीमित है तब मुक्त सिरे पर कुल मरोड़ कोण क्या होगा। कठोरता मापांक $8.4 \times 10^4 \text{ N/mm}^2$ है।



[10 Marks]

Sol.:

Torque on the shaft

$$BC = T$$

∴

$$\text{Torque on the shaft AB} = T - 2T = -T$$

Hence the two shafts are subjected to a torque of same magnitude but of opposite sense.

Hence the shaft BC which is of smaller diameter will be subjected to a greater shear stress than the shaft AB.

Equating the torsional strength of the shaft BC to the external torque, we have,

$$f_s \frac{\pi d^3}{16} = T$$

∴

$$T = \frac{70 \times \pi \times 50^3}{16} = 1718060 \text{ N-mm}$$

Let the twist of the shaft BC be θ_{bc} radians,

∴

$$\frac{T}{J_{bc}} = \frac{C \theta_{bc}}{L_{bc}}$$

∴

$$\theta_{bc} = \frac{1718060}{\left(\frac{\pi \times 50^4}{32}\right)} \times \frac{1800}{8.4 \times 10^4}$$

$$= 0.06 \text{ radian}$$

Similarly the twist θ_{ab} of shaft AB is given by

$$\theta_{ab} = \frac{1718060}{\left(\frac{\pi \times 100^4}{32}\right)} \times \frac{1200}{8.4 \times 10^4}$$

$$= 0.0025 \text{ radian}$$

Since the directions of twist θ_{bc} and θ_{ab} are opposite to each other.

$$\begin{aligned} \text{Net angle of twist of the free end} &= \theta_{bc} - \theta_{ab} \\ &= 0.06 - 0.0025 \\ &= 0.0575 \text{ radian} \\ &= 3^\circ.295 \text{ say } 3^\circ 18' \end{aligned}$$

Ans.

◀ Solution End.

2.(a) A cantilever of length 2 m carries a point load of 20 kN at the free end and another load of 20 kN at its centre. If $E = 10^5 \text{ N/mm}^2$ and $I = 10^8 \text{ mm}^4$ for the cantilever then determine by moment area method, the slope and deflection of the cantilever at the free end.

एक कैंटीलीवर जिसकी लम्बाई 2m है, के मुक्त सिरे पर 20 kN का भार लगा हुआ है। और 20 kN का दूसरा भार केन्द्र पर लगा हुआ है यदि $E = 10^5 \text{ N/mm}^2$ और $I = 10^8 \text{ mm}^4$ हैं तब आघूर्ण क्षेत्रफल विधि की सहायता से कैंटीलीवर बीम के मुक्त सिरे पर स्लोप व विक्षेपण ज्ञात कीजिए।

[20 Marks]

Sol.:

Given, Length, $L = 2\text{m}$

Load at free end, $W_1 = 20 \text{ kN} = 20000 \text{ N}$

Load at centre, $W_2 = 20 \text{ kN} = 20000 \text{ N}$

Value of $E = 10^5 \text{ N/mm}^2$

Value of $I = 10^8 \text{ mm}^4$

First draw the B.M. diagram,

B.M. at B = 0

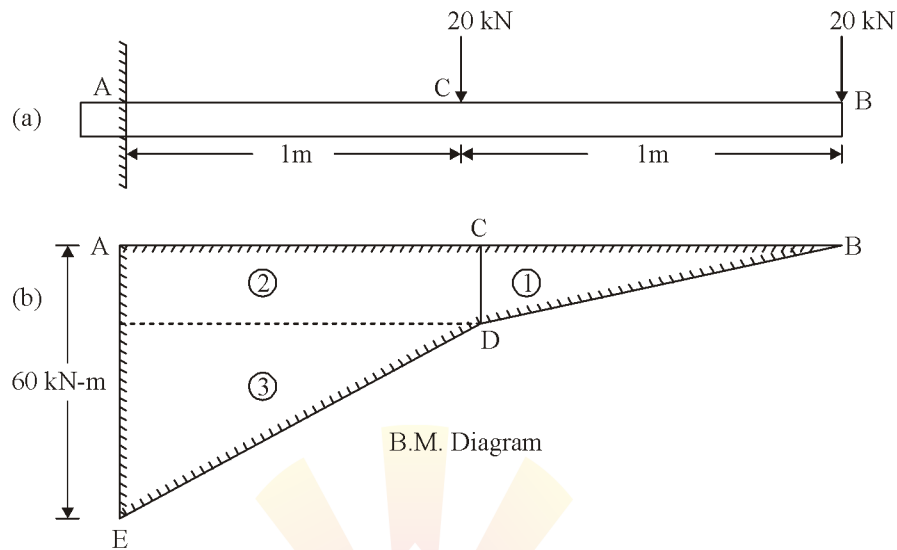
B.M. at C = $-20 \times 1 = -20 \text{ kNm}$

$$= -20 \times 10^3 \times 10^3 \text{ N-mm}$$

B.M. at A = $-20 \times 1 - 20 \times 2$

$$= -60 \text{ kN-m} = -60 \times 10^3 \times 10^3 \text{ N-mm}$$

B.M. diagram is shown in figure



To find the area of B.M. diagram, divide the figure into two triangles and one rectangle.

Now,

$$\begin{aligned} \text{Area } A_1 &= \frac{1}{2} \times CD \times BC = \frac{1}{2} \times 20 \times 1 \\ &= 10 \text{ kNm}^2 = 10 \times 10^3 \times 10^6 \text{ N-mm}^2 \quad (\because \text{m}^2 = 10^6 \text{ mm}^2) \\ &= 10^{10} \text{ N-mm}^2 \end{aligned}$$

Similarly area

$$A_2 = CD \times AC = 20 \times 1 = 20 \text{ kNm}^2$$

and

$$\text{Area } A_3 = \frac{1}{2} \times FD \times EF$$

$$= \frac{1}{2} \times 1 \times 40 = 20 \text{ kN/m}^2$$

\therefore Total area of B.M. diagram,

$$\begin{aligned} A &= A_1 + A_2 + A_3 = 10 + 20 + 20 = 50 \text{ kNm}^2 \\ &= 50 \times 10^3 \times 10^6 \text{ N-mm}^2 \quad (\because \text{m}^2 = 10^6 \text{ mm}^2) \end{aligned}$$

Slope and deflection at the fixed end is zero.

Let

$$\theta_B = \text{Slope at the free end B}$$

Then according to the moment area method

$$\begin{aligned} \theta_B &= \frac{\text{Area of B.M. diagram}}{EI} \\ &= \frac{50 \times 10^3 \times 10^6}{10^5 \times 10^8} = 0.005 \text{ radiSol.} \end{aligned}$$

Let

$$y_B = \text{Deflection at the free end B.}$$

Then according to moment area method,

$$y_B = \frac{A\bar{x}}{EI} \quad \dots(i)$$

Now let us find \bar{x} or $A\bar{x}$.

Then total moment of the bending moment diagram about B is given by

$$\begin{aligned} A \cdot \bar{x} &= A_1\bar{x}_1 + A_2\bar{x}_2 + A_3\bar{x}_3 \\ &= 10 \times \left(\frac{2}{3} \times 1\right) + 20 \times \left(1 + \frac{1}{2}\right) + 20 \times \left(1 + \frac{2}{3} \times 1\right) \\ &= \frac{20}{3} + 30 + \frac{100}{3} = 70 \text{ kNm}^3 \\ &= 70 \times 10^3 \times 10^9 \text{ N-mm}^3 \\ &= 7 \times 10^{13} \text{ Nmm}^3 \quad (\because \text{m}^3 = 10^9 \text{ mm}^3) \\ &= 7 \times 10^{13} \text{ Nmm}^3 \end{aligned}$$

Substituting this value in equation (i), we get

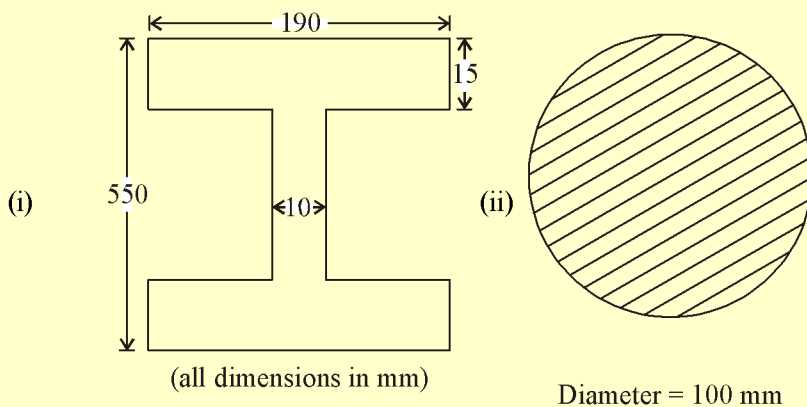
$$y_B = \frac{7 \times 10^{13}}{10^5 \times 10^8} = 7 \text{ mm}$$

Ans.

◀ Solution End.

2.(b) Calculate the moment of Resistance of the given beams. If the maximum stress is 100 N/mm^2 .

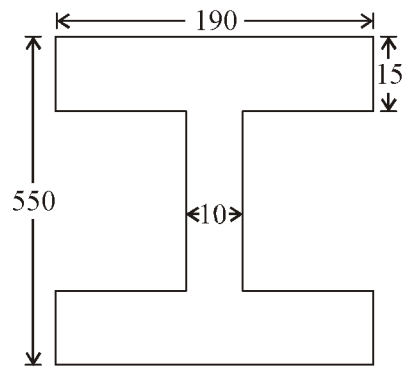
दी गयी बीम के लिए आघूर्ण प्रतिरोध की गणना करें। यदि अधिकतम प्रतिबल 100 N/mm^2 है।



[20 Marks]

Sol.:

(i) For the given beam



(all dimensions in mm)

Since the section is symmetric about both axes, So the Neutral-axes will pass through geometrical-center.

Moment of Inertia of the section.

$$I_{xx} = \frac{190 \times 550^3}{12} - \frac{(190 - 10) \times 520^3}{12}$$

$$I_{xx} = 525150833.3 \text{ mm}^4$$

Bending stress

$$f = \frac{M}{I} \cdot y$$

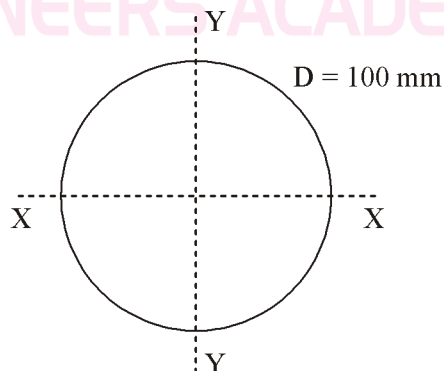
So,

$$M = \frac{f \cdot I}{y}$$

So, moment of Resistance of the Beam is

$$M = \frac{100 \times 525150833.3 \times 10^{-6}}{\left(\frac{550}{2}\right)} = 190.96 \text{ kN-m}$$

(ii)



$$\text{Moment of Inertia } I_{xx} = \frac{\pi}{64} D^4$$

$$I_{xx} = \frac{\pi}{64} \times (100)^4 = 4908738.52 \text{ mm}^4$$

Since bending stress

$$f = \frac{M}{I} \cdot y$$

So, moment of resistance

$$M = \frac{f \cdot I}{y} = \frac{100 \times 4908738.52}{\left(\frac{100}{2}\right)}$$

Given maximum stress

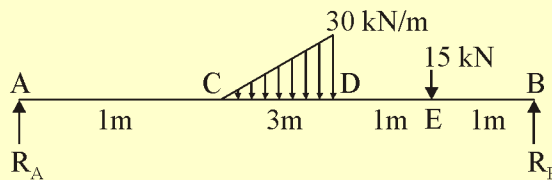
$$f = 100 \text{ N/mm}^2$$

$$M = 9.81 \text{ kN-m}$$

◀ Solution End.

2.(c) Draw shear force and bending moment diagrams for the loaded beam shown in figure.

आकृति में दिखाए गए भार धरन के लिए कर्तन बल आरेख और बंकन आघूर्ण आरेख खींचें (दर्शाये)।



[20 Marks]

Sol.:

$$R_A + R_B - \frac{1}{2} \times 30 \times 3 - 15 = 0$$

$$R_A + R_B = 60 \text{ kN}$$

Moment at 'A'

$$R_B \times 6 - 15 \times 5 - \frac{1}{2} \times 30 \times 3 \left[\frac{2L}{3} + 1 \right] = 0$$

$$R_B \times 6 = 75 + 45 \left[\frac{2 \times 3}{3} + 1 \right]$$

$$R_B = \frac{210}{6} = 35 \text{ kN}, R_A = 60 - 35 = 25 \text{ kN}$$

$$SF_A = 25 \text{ kN}$$

$$SF_C = 25 \text{ kN}$$

$$SF_D = 25 - \frac{1}{2} \times 30 \times 3 = -20 \text{ kN}$$

$$SF_E = -20 - 15 = -35 \text{ kN}$$

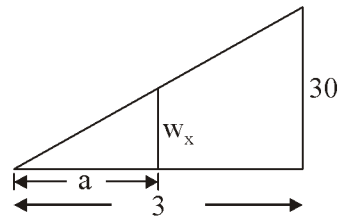
$$SF_B = -35 + 35 = 0 \text{ kN}$$

Let SF be zero at a distance 'a' from C

$$F_x = 25 - \frac{1}{2} \times 10a \times a = 0$$

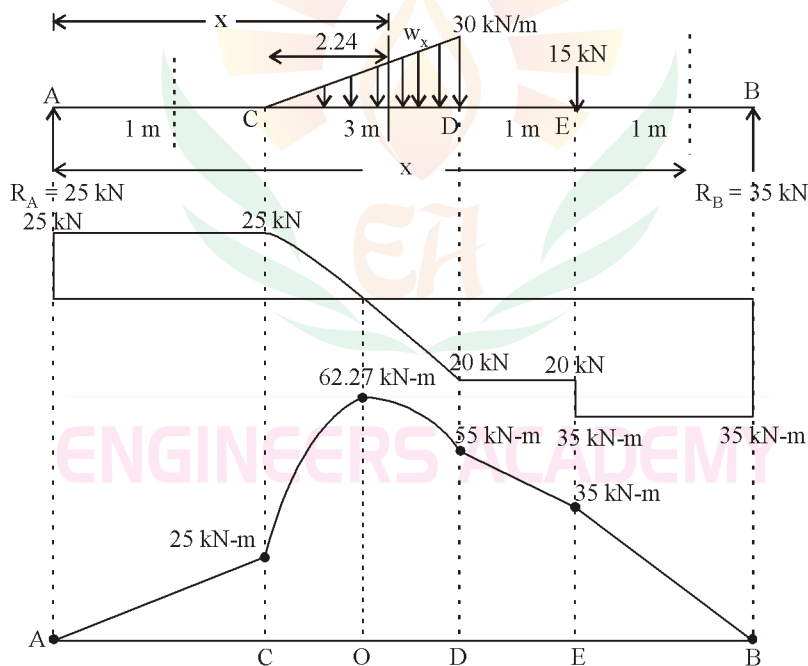
$$5a^2 = 25$$

$$a = 2.24 \text{ m}$$



$$\frac{w_x}{a} = \frac{30}{3}$$

$$w_x = 10 a$$



$$M_x = 25x - \frac{1}{2} \times 30 \times 3 \left(x - \frac{2L}{3} - 1 \right) - 15(x - 5)$$

$$M_x = 25x - 45(x - 3) - 15(x - 5)$$

$$x = 0, M_A = 0 \text{ kN-m}$$

$$x = 1, M_C = 25 \times 1 = 25 \text{ kN-m}$$

$$x = 4, M_D = 25 \times 4 - 45(4 - 3) = 55 \text{ kN-m}$$

$$x = 5, M_E = 25 \times 5 - 45(5-3) - 15(5-5) \\ = 35 \text{ kN-m}$$

$$x = 6, M_B = 0 \text{ kN-m}$$

Maximum B.M. :

At $x = 3.24$

$$M_x = 25x - \frac{1}{2} \times w_x \times a \left(\frac{a}{3} \right)$$

$$= 25x - \frac{1}{2} \times 10a \times a \left(\frac{a}{3} \right)$$

$$x = 3.24$$

$$(M)_{\max} = 25 \times 3.24 - \frac{1}{2} \times 10 \times (2.24)^2 \times \left(\frac{2.24}{3} \right) \\ = 62.27 \text{ kN-m}$$

◀ Solution End.

3.(a) Determine Euler's crippling load for an I-section joints $300 \text{ mm} \times 150 \text{ mm} \times 20 \text{ mm}$ and 5 m long which is used as a strut with both ends fixed. Take $E = 2 \times 10^5 \text{ N/mm}^2$

एक I-section जोड़ों के लिए आयलर के किपलिंग लोड की गणना करें, जो $300 \text{ mm} \times 150 \text{ mm} \times 20 \text{ mm}$ और 5 m लम्बा है। जो दोनों आबद्ध सिरों के साथ एक अकड़ (strut) के रूप में उपयोग किया जाता है। $E = 2 \times 10^5 \text{ N/mm}^2$ है।

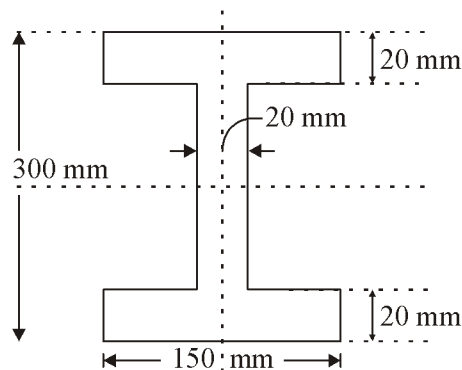
[25 Marks]

Sol.:

Given :

$$l = 5 \text{ m}$$

$$E = 2 \times 10^5 \frac{\text{N}}{\text{mm}^2}$$



Moment of inertia about X-X axis

$$\begin{aligned} I_{xx} &= \frac{1}{12}150 \times 300^3 - \frac{1}{12} \times 130 \times 260^3 \\ &= 3.375 \times 10^8 - 1.904 \times 10^8 \\ &= 1.471 \times 10^8 \text{ mm}^4 \end{aligned}$$

I_{yy} = M.I. about yy axis

$$\begin{aligned} &= 2 \times \frac{1}{12} \times 20 \times 150^3 + \frac{1}{12} \times 260 \times 20^3 \\ &= 11250000 + 173333.33 \\ &= 11423333 = 1.142 \times 10^7 \text{ mm}^4 \end{aligned}$$

Least moment of inertia is I_{yy}

$$I = 1.142 \times 10^7 \text{ mm}^4$$

Column with both end fixed

Effective Length

$$(L) = \frac{l}{2} = \frac{5000}{2} = 2500 \text{ mm}$$

$$\begin{aligned} P &= \frac{\pi^2 EI}{L^2} = \frac{\pi^2 \times 2 \times 10^5 \times 1.142 \times 10^7}{(2500)^2} \\ &= 3606748.2 \text{ N} \\ &= 3606.7 \text{ kN} \end{aligned}$$

◀ Solution End.

3.(b) A solid circular shaft of 100 mm diameter is to transmit a maximum torque of 70620 Nm. Find the shear stress at :

100 mm व्यास का ठोस वृताकार शाफ्ट 70620 Nm का अधिकतम बल आघूर्ण संचारित करता है। तो कर्तन प्रतिबल ज्ञात करो।

(i) Centre of shaft

शाफ्ट के केन्द्र पर

(ii) Outermost fibre of shaft

शाफ्ट के बाहरीतम फाइबर पर

[15 Marks]

Sol.:

Given

$$D = 100 \text{ mm}$$

$$T = 70620 \text{ N-m}$$

Polar moment of Inertia

$$I_p = \frac{\pi}{32} D^4$$

$$= \frac{\pi}{32}(100)^4 = 9817477 \text{ mm}^4$$

(i) Shear stress at the centre of the shaft,

We know
$$\frac{T}{I_p} = \frac{\tau}{R}$$

at the centre
$$R = 0$$

$$\frac{T}{I_p} = \frac{\tau}{0}$$

$$\tau = 0$$

(ii) Shear stress at the outer most fibre of shaft.

At the outer fibre
$$R = \frac{D}{2} = \frac{100}{2} = 50 \text{ mm}$$

$$\frac{T}{I_p} = \frac{\tau}{R}$$

$$\frac{70620 \times 10^3}{9817477} = \frac{\tau}{50}$$

$$\tau = \frac{70620 \times 10^3 \times 50}{9817477}$$

$$\tau = 359.66 \text{ N/mm}^2$$

◀ Solution End.

3.(c) The tensile test has been performed on a 50 mm long steel tube having 18 mm external dia. and 12 mm internal dia. The axial load of 2kN produced a stretch of 3.36×10^{-3} mm and a lateral contraction of the outer dia of 3.62×10^{-4} mm. Calculate young's modulus, poisson's ratio and the bulk modulus for the material.

12mm आन्तरिक व 18mm बाह्य व्यास और 50mm लम्बी steel tube पर तनाव परीक्षण किया गया। जिसमें 2kN का अक्षीय भार 3.36×10^{-3} mm का विरूपण उत्पन्न करता है और 3.62×10^{-4} के बाह्य व्यास पर पार्श्व contraction उत्पन्न करता है! पदार्थ के लिए यंग मापांक, पॉइजन अनुपात और आयतन मापांक की गणना कीजिए!

[20 Marks]

Sol.:

Given data

$$l = 50 \text{ mm } D_e = 18 \text{ mm } D_i = 12 \text{ mm}$$

$$P = 2 \text{ kN} = 2 \times 10^3 \text{ N}$$

$$\Delta l = 3.36 \times 10^{-3} \text{ mm}$$

$$\Delta d = 3.62 \times 10^{-4} \text{ mm}$$

$$E = ? \quad \mu = ? \quad K = ?$$

$$\text{Stress } (\sigma) = \frac{P}{A} = \frac{2 \times 10^3 \text{ N}}{\frac{\pi}{4}(18-12)^2} = 70.7 \frac{\text{N}}{\text{mm}^2}$$

$$\text{Strain } (\varepsilon) = \frac{\Delta l}{L} = \frac{3.36 \times 10^{-3} \text{ mm}}{50 \text{ mm}} = 6.72 \times 10^{-5}$$

Young's modulus

$$E = \frac{\sigma}{\varepsilon} = \frac{70.7}{6.72 \times 10^{-5}} = 10.52 \times 10^5 \frac{\text{N}}{\text{mm}^2}$$

lateral strain

$$= \frac{\delta d}{d} = \frac{3.62 \times 10^{-4}}{18} = 2.01 \times 10^{-5}$$

Poisson's ratio

$$\mu = \frac{\delta d / d}{\Delta l / l} = \frac{2.01 \times 10^{-5}}{6.72 \times 10^{-5}}$$

$$= 0.2993$$

$$E = 3K(1-2\mu)$$

$$10.52 \times 10^5 = 3K(1 - 2 \times 0.2993)$$

$$K = 8.74 \times 10^5 \text{ N/mm}^2$$

◀ Solution End.

4.(a) Determine the diameter of a solid shaft which will transmit 300 kW at 250 rpm. The maximum shear stress should not exceed 30 N/mm² and twist should not be more than 1° in a shaft length of 2m. Take modulus of rigidity = 1 × 10⁵ N/mm².

एक टोस शाफ्ट का व्यास ज्ञात कीजिए जो 250 rpm पर 300 kW पावर ट्रांसमिट करता है। 2m लम्बाई की शाफ्ट में अधिकतम अपरूपण प्रतिबल 30 N/mm² से ज्यादा नहीं होना चाहिए और टिबस्ट 1° से ज्यादा नहीं होना चाहिए। कठोरता मापांक = 1 × 10⁵ N/mm² है

[20 Marks]

Sol.:

Given :

Power transmitted,

$$P = 300 \text{ kW} = 300 \times 10^3 \text{ W}$$

Speed of the shaft,

$$N = 250 \text{ rpm}$$

Maximum shear stress,

$$\tau = 30 \text{ N/mm}^2$$

Twist in shaft,

$$\theta = I^\circ = \frac{\pi}{180} = 0.01745 \text{ radian}$$

Length of shaft,

$$L = 2 \text{ m} = 2000 \text{ mm}$$

Modulus of rigidity,

$$C = 1 \times 10^5 \text{ N/mm}^2$$

Let

D = Diameter of the shaft,

Power is given by the relation,

$$P = \frac{2\pi NT}{60}$$

or

$$300 \times 10^5 = \frac{2\pi \times 250 \times T}{60}$$

$$T = \frac{300 \times 10^3 \times 60}{2\pi \times 250}$$

$$= 11459.1 \text{ N-m}$$

$$= 11459.1 \times 10^3 \text{ N-mm}$$

(i) Diameter of the shaft when maximum shear stress,

$$\tau = 30 \text{ N/mm}^2$$

Maximum torque transmitted by a solid shaft is given by equation as

$$T = \frac{\pi}{16} \times \tau \times D^3$$

$$11459100 = \frac{\pi}{16} \times 30 \times D^3$$

$$D = \left(\frac{16 \times 11459100}{\pi \times 30} \right)^{1/3}$$

$$= 124.5 \text{ mm}$$

(ii) Diameter of shaft when twist should not be more than 1° .

Using equation

$$\frac{T}{J} = \frac{C\theta}{L}$$

where J = Polar moment of inertia of solid shaft

$$= \frac{\pi}{32} D^4$$

$$\frac{11459100}{\frac{\pi}{32} D^4} = \frac{10^5 \times 0.01745}{2000}$$

$$D^4 = \frac{32 \times 2000 \times 11459100}{10^5 \times \pi \times 0.01745}$$

$$= 13377.81 \times 10^4$$

$$D = (13377.81 \times 10^4)^{1/4} = 107.5 \text{ mm} \quad \dots(ii)$$

The suitable diameter of the shaft is the greater of the two values given by equations (i) and (ii)

\therefore Diameter of the shaft = 124.5 mm say 125 mm.

◀ Solution End.

4.(b) Write the difference between force method and displacement method of analysis. Also write the examples of force method and displacement method.

विश्लेषण की बल विधि और विस्थापन विधि में अंतर लिखिए। बल विधि और विस्थापन विधि के उदाहरण भी लिखिए।

[10 Marks]

Sol.:

S.No.	Force Method or Compatibility Method Or Flexibility Method	Displacement Method Or Equilibrium Method Or Stiffness method
1.	Number of additional unknowns in this case are equals to D_s .	Number of additional unknowns in this case are equals to D_k .
2.	Unknowns in this case are forces (Rxn, S.F., M.B.)	Unknowns in this case are displacement (Δ, ϕ)
3.	Force-displacement equations are written and solution for unknown forces is obtained from compatibility equations.	Force-displacement equations are written and solution for unknown displacement is obtained from equilibrium equations.
4.	Once unknown forces are known, then remaining forces are found using equilibrium equations.	Once unknown displacement are known then remaining forces are found using compatibility and load displacement equations.
5.	Force method of analysis of indeterminate structure is suitable when degree of static indeterminacy is less than degree of Kinematic indeterminacy.	Displacement method is suitable when degree of kinematic indeterminacy is less than degree of static indeterminacy.

Examples of force Methods

- Castigliano's theorem
- Strain energy method
- Virtual work method
- Claperon's three moment equation
- Column analogy method
- Flexibility matrix method

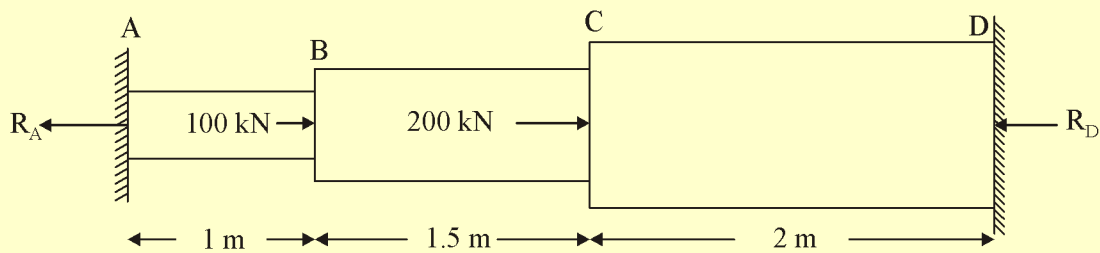
Examples of Displacement Methods

- Moment distribution method
- Slope deflection method
- Stiffness matrix method
- Kani's method

◀ Solution End.

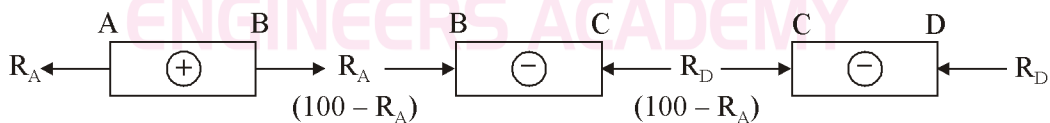
4.(c) A bar of varying sections, rigidly fixed at A and D subjected to axial forces. Determine the force in each portion of the bar and the displacements of point B and C. Portion AB, of length 1 m has area of cross-section 2000 mm^2 , portion BC of length 1.5 m has area of cross-section of 3000 mm^2 , while portion CD has a length of 2m and area of cross-section of 4000 mm^2 . Take $E = 200 \text{ kN/mm}^2$.

एक बार जिसका सेक्शन परिवर्तनीय जो कि A और D पर कठोर रूप से फिक्स हैं तथा अक्षीय बलों के अधीन है। बार के प्रत्येक भाग में बल और बिंदु B तथा C पर विस्थापन का निर्धारण करे। भाग AB जिसकी लम्बाई 1m है तथा क्रॉस सेक्शन क्षेत्रफल 2000 mm^2 है और भाग BC जिसकी लम्बाई 1.5 मी है तथा C/S क्षेत्रफल 3000 mm^2 है तथा भाग CD जिसकी लम्बाई 2m है और C/S क्षेत्रफल 4000 mm^2 है। $E = 200 \text{ kN/mm}^2$

**[15 Marks]****Sol.:**

$$\Sigma F_x = 0$$

$$R_A + R_D = 300 \text{ kN} \quad \dots(i)$$



Support A and D are rigidly fixed

$$\Delta_{AD} = 0$$

$$\Delta_{AB} - \Delta_{BC} - \Delta_{CD} = 0$$

$$\frac{R_A \times 1 \times 10^3}{2000 \times E} - \frac{(100 - R_A) \times 1.5 \times 10^3}{3000 \times E} - \frac{R_D \times 2 \times 10^3}{4000 \times E} = 0$$

$$R_D = 300 - R_A \text{ from equation (1)}$$

$$0.5 R_A - 50 + 0.5 R_A - \left(\frac{300 - R_A}{2} \right) = 0$$

$$1.5 R_A - 200 = 0$$

$$R_A = +133.33 \text{ kN (tensile)}$$

$$R_D = 166.66 \text{ kN (compression)}$$

Force in member AB = 133.33 kN tensile

Force in member BC = 33.33 kN (tensile)

Force in member CD = 166.66 kN (compressive)

$$\text{Elongation in AB} = \frac{PL}{AE} = \frac{133.33 \times 10^3 \times 1 \times 10^3}{2000 \times 2 \times 10^5}$$

$$\Delta_B = 0.333 \text{ mm } (\rightarrow)$$

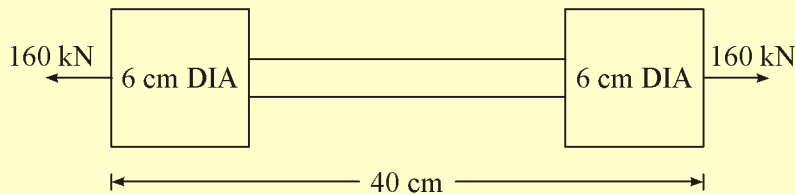
$$\text{Contraction in CD} = \frac{PL}{AE} = \frac{166.66 \times 10^3 \times 2 \times 10^3}{4000 \times 2 \times 10^5}$$

$$\Delta_D = 0.4166 \text{ mm } (\rightarrow)$$

◀ Solution End.

4.(d) The bar shown in figure is subjected to a tensile load of 160 kN. If the stress in the middle portion is limited to 150 N/mm². Determine the diameter of the middle portion. Find also the length of the middle portion, if the total elongation of the bar is to be 0.2 mm. Young's modulus is given as equal to 2.1×10^5 N/mm².

एक बार पर चित्र में दिखाये अनुसार 160 kN का तन्व बल लगा हुआ है यदि मध्य भाग में प्रतिबल 150 N/mm² तक सीमित है तब मध्य भाग के व्यास की गणना करें तथा मध्य भाग की लम्बाई की गणना करें। यदि बार का कुल विस्तार 0.2 mm है। यंग माड्यूलस 2.1×10^5 N/mm² दिया गया है।



[15 Marks]

Sol.:

Given : Tensile load, $P = 160 \text{ kN} = 160 \times 10^3 \text{ N}$

Stress in middle portion, $\sigma_2 = 150 \text{ N/mm}^2$

Total elongation, $dL = 0.2 \text{ mm}$

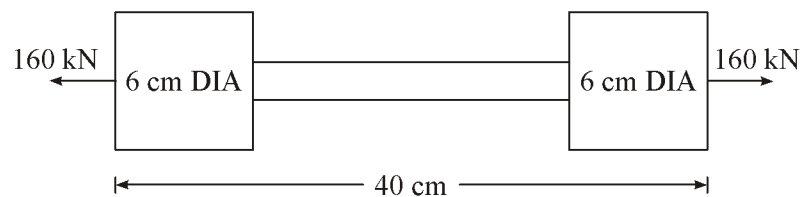
Total length of the bar, $L = 40 \text{ cm} = 400 \text{ mm}$

Young's modulus, $E = 2.1 \times 10^5 \text{ N/mm}^2$

Diameter of both end portions, $D_1 = 6 \text{ cm} = 60 \text{ mm}$

∴ Area of cross-section of both end portions,

$$A_1 = \frac{\pi}{4} \times 60^2 = 900 \pi \text{ mm}^2$$



Let

D_2 = Diameter of the middle portion

L_2 = Length of middle portion in mm

∴ Length of both end portions of the bar,

$$L_1 = (400 - L_2) \text{ mm}$$

Using equation (1.1), we have

$$\text{Stress} = \frac{\text{Load}}{\text{Area}}$$

For the middle portion, we have

$$\sigma_2 = \frac{P}{A_2} \quad \text{where } A_2 = \frac{\pi}{4} D_2^2$$

or

$$150 = \frac{160000}{\frac{\pi}{4} D_2^2}$$

∴

$$D_2^2 = \frac{4 \times 160000}{\pi \times 150} = 1358 \text{ mm}^2$$

$$D_2 = \sqrt{1358} = 36.85 \text{ mm}$$

∴ Area of cross-section of middle portion,

$$A_2 = \frac{\pi}{4} 36.85^2 = 1066 \text{ mm}^2$$

Now using equation, we get

Total extension,

$$dL = \frac{P}{E} \left[\frac{L_1}{A_1} + \frac{L_2}{A_2} \right]$$

or

$$0.2 = \frac{160000}{2.1 \times 10^5} \left[\frac{(400 - L_2)}{900\pi} + \frac{L_2}{1066} \right]$$

$$[\because L_1 = (400 - L_2) \text{ and } A_2 = 1066]$$

or

$$\frac{0.2 \times 2.1 \times 10^5}{160000} = \frac{(400 - L_2)}{900\pi} + \frac{L_2}{1066}$$

or

$$0.2625 = \frac{1066(400 - L_2) + 900\pi L_2}{900\pi \times 1066}$$

or

$$0.2625 \times 900 \pi \times 1066 = 1066 \times 400 - 1066 L_2 + 900\pi \times L_2$$

or

$$791186 = 426400 - 1066 L_2 + 2827 L_2$$

or

$$364786 - 426400 = L_2(2827 - 1066)$$

or

$$364786 = 1761 L_2$$

∴

$$\begin{aligned} L_2 &= \frac{364786}{1761} \\ &= 207.14 \text{ mm} \\ &= 20.714 \text{ cm} \end{aligned}$$

◀ Solution End.

5.(a) Write the difference between statically determinate and indeterminate structure.

स्थिर रूप से निर्धारित व अनिर्धारित संरचना के बीच अंतर लिखिए।

[10 Marks]

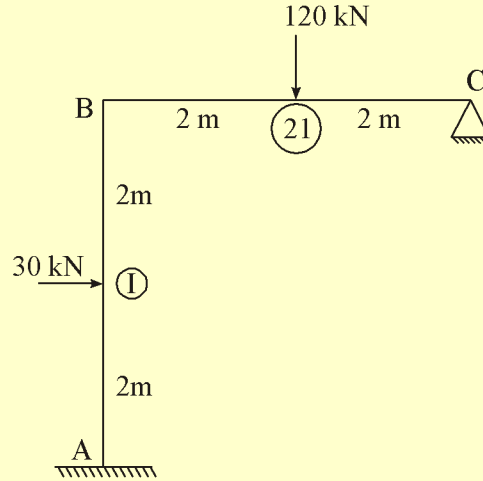
Sol.:

S.No.	Determinate Structures	Indeterminate Structures
1.	Equilibrium conditions are fully adequate to analyse the structure.	Conditions of equilibrium are not adequate to fully analyse the structure.
2.	Bending moment or shear force at any section is independent of the material property of the structure.	Bending moment or shear force at any section depends upon the material property.
3.	The bending moment or shear force at any section is independent of the cross-section or moment of inertia.	The bending moment or shear force at any section depends upon the cross-section or moment of inertia.
4.	Temperature variations do not cause stresses.	Temperature variations cause stresses.
5.	No stresses are caused due to lack of fit.	Stresses are caused due to lack of fit.
6.	Extra conditions like compatibility of displacements are not required to analyse the structure.	Extra conditions like compatibility of displacements are required to analyse the structure along with the equilibrium equations.

◀ Solution End.

5.(b) Analysis the frame shown in figure below using slope deflection method.

चित्र में दिखाये गये फ्रेम का ढलान विक्षेपण विधि से विश्लेषण करें।



[25 Marks]

Sol.:

Here, there is no horizontal sway as member BC is fixed at C. (fixed hinge). Thus, degrees of freedom = 2, i.e. θ_B and θ_C while $\theta_A = 0$.

Fixed end moments

$$M_{FAB} = -\frac{PL}{8} = -30 \times \frac{4}{8}$$

$$= -15 \text{ kN-m}$$

$$M_{FBA} = 15 \text{ kN-m}$$

$$M_{FBC} = -120 \times \frac{4}{8} = -60 \text{ kNm}$$

$$M_{FCB} = 60 \text{ kN-m}$$

Slope deflection equations:

$$M_{AB} = \left[\frac{2EI(2\theta_A + \theta_B)}{4} \right] - 15;$$

$$M_{BA} = \left[\frac{2EI(\theta_A + 2\theta_B)}{4} \right] + 15$$

$$M_{BC} = \left[\frac{2 \times 2EI(2\theta_B + \theta_C)}{4} \right] - 60;$$

$$M_{CB} = \left[\frac{2 \times 2EI(\theta_B + 2\theta_C)}{4} \right] + 60$$

Equilibrium Equations

At hinge; $M_{CB} = 0$, giving $\theta_B + 2\theta_C = -\frac{60}{EI}$... (i)

At point B, $M_{BA} + M_{BC} = 0$ gives

$$EI\theta_B + 15 + 2EI\theta_B + EI\theta_C - 60 = 0$$

On simplification, we get, $3\theta_B + \theta_C = \frac{45}{EI}$... (ii)

Solution of equations

On solving equations (i) and (ii), we get,

$$\theta_B = \frac{30}{EI} \text{ (clockwise) and } \theta_C = \frac{-45}{EI} \text{ (anticlockwise)}$$

On substitution these values, final moments are as follows:

$$M_{AB} = \left[\frac{2EI \left(\frac{1}{EI} \right) (30)}{4} \right] - 15 = 0$$

$$M_{BA} = \left[\frac{2EI \left(\frac{1}{EI} \right) (2 \times 30)}{4} \right] + 15 = 45 \text{ kN-m}$$

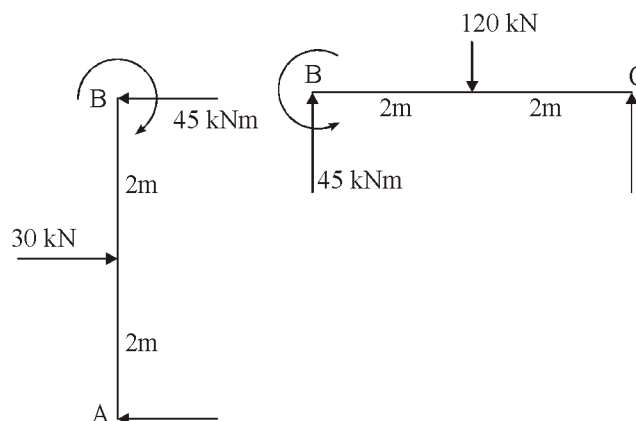
$$M_{BC} = \left[\frac{2 \times 2EI \left(\frac{1}{EI} \right) (2 \times 30 - 45)}{4} \right] - 60$$

$$= -45 \text{ kN-m}$$

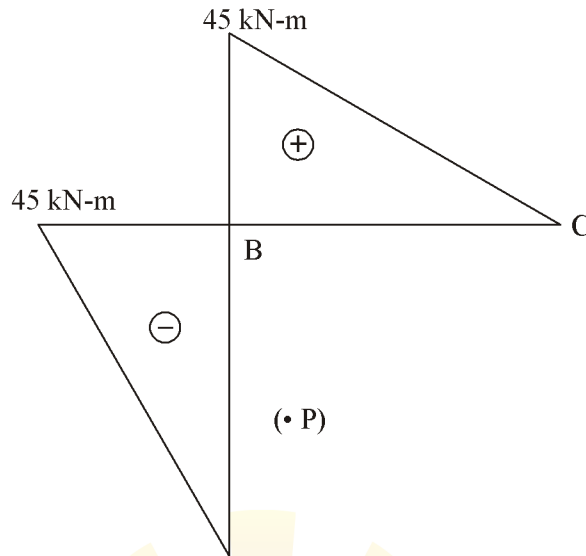
$$M_{CB} = \left[\frac{4EI \left(\frac{1}{EI} \right) (30 - 2 \times 45)}{4} \right] + 60 = 0$$

Here, although A is fixed, incidentally $M_{AB} = 0$.

The free body diagram for the frame is as shown below.



Here the end moment diagram is



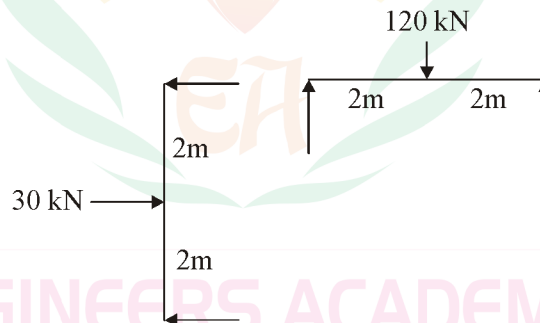
Note that BM has been plotted on tension side.

To give the sign to the BMD, we sit inside the frame and give sign as per bending moment seen from point P.

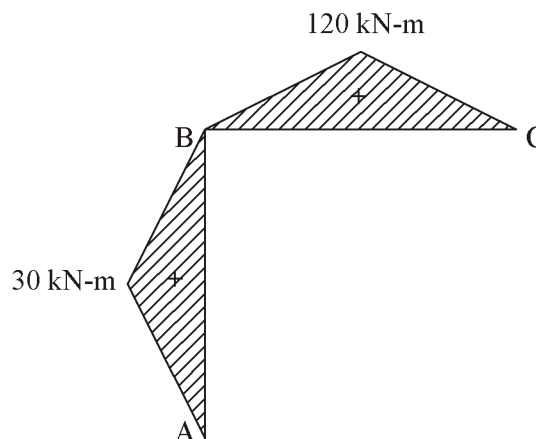
BM as seen from 'P' for portion AB is hogging hence (-) ve sign is given.

BM as seen from P for portion BC is hogging hence (-) ve sign is given.

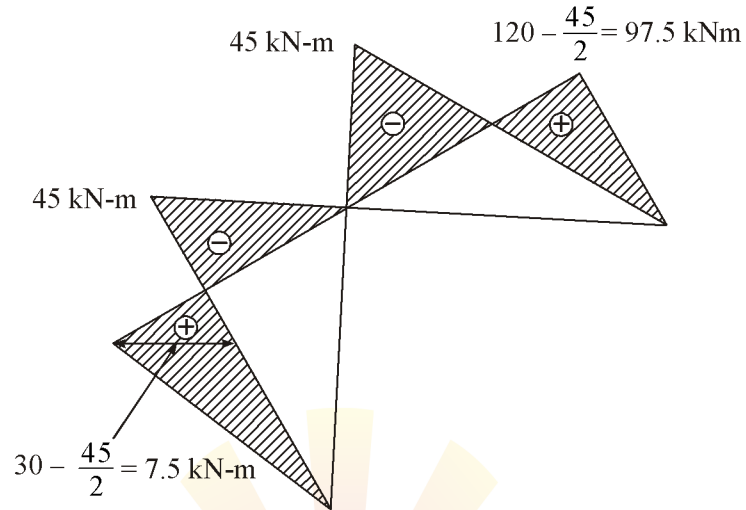
2. Free moment diagram is drawn for the loading condition shown below.



The free moment diagram with sign is as given below.



3. The free moment diagram and the end moment diagrams are superimposed as shown below.

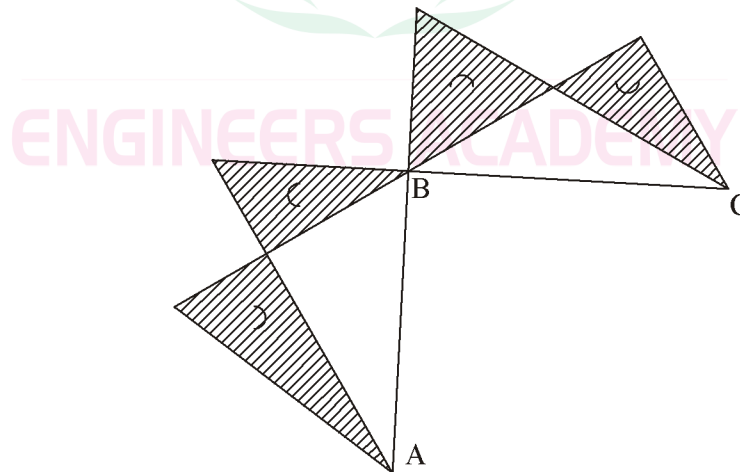


This is the final bending moment diagram.

Note that instead of giving (+)ve and (-)ve sign, we can also given symbols like \cap or \cup .

This symbol shows that tension is produced on β -side.

Thus the BMD can also be shown as



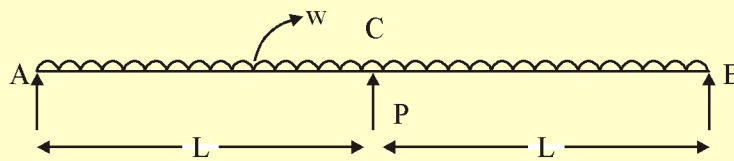
◀ Solution End.

5.(c) A beam of uniform section and of length $2L$ is freely supported by rigid supports at its ends and by an elastic prop at its centre. If the the prop deflects by an amount λ times the load it carries and if the beam carries a

total distributed load of W , show that the load carried by the prop is $\frac{5W}{8\left(1+\frac{6IE\lambda}{L^3}\right)}$

समान खण्ड और लम्बाई $2L$ का एक बीम स्वतंत्र रूप से सिरों पर कठोर समर्थन से समर्थित हैं और इसके केन्द्र पर एक लोचदार प्रोप द्वारा समर्थित है यदि प्रोप, बीम पर लगे लोड का λ गुणा विक्षेपित होता है और यदि बीम पर कुल

UDL W लगा हुआ है तब दिखाए की प्रोप लोड $\frac{5W}{8\left(1+\frac{6IE\lambda}{L^3}\right)}$ होगा।



[25 Marks]

Sol.:

Let, the prop reaction be = P

load on beam = W

UDL = $\frac{W}{2L}$ per unit length

Reaction at ends = $R = \frac{1}{2}(W - P)$

The deflection at the prop is given by

$$\delta_c = \frac{\partial u}{\partial P} = \frac{1}{EI} \int M_x \frac{\partial M_x}{\partial P} dx$$

Since the prop deflects by an amount λ times the load it carries, we have $\delta_c = -P\lambda$

Hence, $-P\lambda = \frac{1}{EI} \int M_x \frac{\partial M_x}{\partial P} dx$

$$-P = \frac{1}{\lambda EI} \int M_x \frac{\partial M_x}{\partial P} dx \quad \dots (i)$$

For any section distant x from A,

$$M_x = \frac{1}{2}(W - P)x - \frac{Wx^2}{2Lx \times 2}$$

$$\frac{\partial M_x}{\partial P} = \frac{x}{2}$$

Put this value in equation (i)

$$-P = \frac{2}{\lambda EI} \int_0^L \left\{ \frac{1}{2}(W-P)x - \frac{Wx^2}{2Lx \times 2} \right\} \left\{ -\frac{x}{2} \right\} dx$$

$$\frac{P\lambda EI}{2} = \left[\frac{1}{12}(W-P)x^3 - \frac{Wx^4}{32L} \right]_0^L$$

$$\frac{P\lambda EI}{2} = \frac{WL^3}{12} - \frac{PL^3}{12} - \frac{WL^3}{32} = \frac{5WL^3}{96} - \frac{PL^3}{12}$$

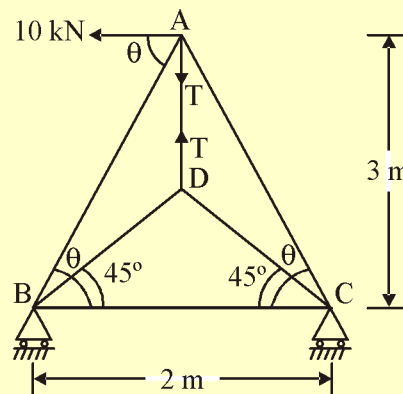
$$P \left(\frac{6EI\lambda}{L^3} + 1 \right) = \frac{5W}{8}$$

$$P = \frac{5W}{8 \left(1 + \frac{6EI\lambda}{L^3} \right)}$$

◀ Solution End.

6.(a) In the frame work shown in figure, the member AB, BC and CA have area of c/s '2a' and the member DA, DB and DC have area of c/s 'a'. find the force in the member DA due to a load 10 kN applied horizontally at A.

आकृति में दिखाये गये चित्र में सदस्य AB, BC और CA का c/s क्षेत्रफल '2a' है और सदस्य DA, DB और DC का c/s क्षेत्रफल 'a' है। A पर क्षैतिज रूप से लगाये गये 10 kN भार के कारण सदस्य DA में बल का पता लगाये।



[25 Marks]

Sol.:

$$M = 6, j = 4, r = 3$$

$$\begin{aligned} D_{si} &= m - (2j - 3) \\ &= 6 - (2 \times 4 - 3) \\ &= 6 - 5 = 1 \end{aligned}$$

Frame is one degree redundant.

AD as redundant, replace it by tensile force T at the joint A and D. Then

$$\frac{\partial U}{\partial T} = 0 = \Sigma P \frac{\partial P}{\partial T} \cdot \frac{L}{A} = 0$$

At the joint D, resolving horizontally,

$$P_{BD} = P_{DC}$$

Resolving vertically

$$P_{BD} \cos 45^\circ + P_{DC} \cos 45^\circ = T$$

$$P_{BD} = P_{DC} = \frac{T}{\sqrt{2}}$$

At the joint A, resolving vertically

$$P_{AB} \sin \theta = T + P_{AC} \sin \theta$$

... (i)

Horizontally

$$P_{AB} \cos \theta + P_{AC} \cos \theta = 10$$

... (ii)

$$\sin \theta = \frac{3}{\sqrt{10}} = 0.948$$

$$\cos \theta = \frac{1}{\sqrt{10}} = 0.316$$

Solving (1) and (2), we get

$$P_{AB} = 0.527 T + 15.85 \text{ (comp)}$$

$$P_{AC} = 15.85 - 0.27 T \text{ (tension)}$$

Resolving horizontally at B

$$P_{BC} = P_{AB} \cos \theta - P_{BD} \cos 45^\circ$$

$$P_{BC} = (0.527T + 15.85)0.316 - \frac{T}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$$

$$P_{BC} = -0.344 T + 5 \text{ (tension)}$$

Member	Length	Area	P	$\partial P / \partial T$	$\left(P \cdot \frac{\partial P}{\partial T} \cdot \frac{L}{A} \right)_a$
AB	3.16	2A	$-(.527T + 15.85)$	-0.527	$0.439T + 13.2$
AC	3.16	2A	$15.85 - 0.527T$	-0.527	$-13.2 + 0.439T$
BC	2.0	2A	$-0.334T + 5$	-0.334	$0.112T - 1.67$
BD	$\sqrt{2}$	A	$T / \sqrt{2}$	$1 / \sqrt{2}$	$0.707T$
CD	$\sqrt{2}$	A	$T / \sqrt{2}$	$1 / \sqrt{2}$	$0.707T$
AD	2.0	A	T	+1	+2T
				Sum	$4.404T - 1.67$

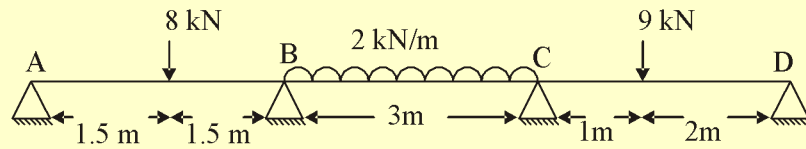
$$\therefore \Sigma P \cdot \frac{\partial P}{\partial T} \cdot \frac{L}{A} = 0 = \frac{1}{A} (4.404T - 1.67)$$

$$T = 0.379 \text{ kN}$$

◀ Solution End.

6.(b) A horizontal beam ABCD is carried on hinged supports and is continuous over three equal spans each of 3m. All the supports are initially at the same level. The beam is loaded as shown in figure. Plot the bending moment diagram, if the support A settle by 10 mm, B settle by 30 mm and C settle by 20 mm. The moment of inertia of the whole beam is $2.4 \times 10^6 \text{ mm}^4$ unit, $E = 2 \times 10^5 \text{ N/mm}^2$.

क्षैतिज बीम ABCD जो कि हिन्ज्ड सपोर्ट पर समर्थित है और 3 मीटर के प्रत्येक तीन बराबर स्पान पर निरंतर होता है सभी सपोर्ट शुरु में समान स्तर पर है। बीम चित्र में दिखायी है। बंकन आघूर्ण चित्र बनाये यदि सपोर्ट A, 10 mm सेटल होता है, B, 30 mm और C, 20 mm सेटल होता है। सम्पूर्ण बीम का जडत्व आघूर्ण $2.4 \times 10^6 \text{ mm}^4$ है और $E = 2 \times 10^5 \text{ N/mm}^2$ है।



[25 Marks]

Sol.:

Span AB sinks by 20 mm

$$M_{FAB} = \frac{-Wl}{8} - \frac{6EI\delta}{L^2}$$

$$= \frac{-8 \times 3}{8} - \frac{6 \times 2 \times 10^5 \times 2.4 \times 10^6 \times 20 \times (10^{-6})}{(3000)^2}$$

$$= -3 - 6.4 = -9.4 \text{ kN-m}$$

$$M_{FBA} = +\frac{Wl}{8} - \frac{6EI\delta}{L^2}$$

$$= +3 - 6.4 = -3.4 \text{ kN-m}$$

Span BC sinks $30 - 20 = 10 \text{ mm} (\downarrow)$

$$M_{FBC} = -\frac{wl^2}{12} + \frac{6EI\delta}{L^2}$$

$$= \frac{-2 \times 3^2}{12} + \frac{6 \times 2 \times 10^5 \times 2.4 \times 10^6 \times 10 \times 10^{-6}}{(3000)^2}$$

$$= -1.5 + 3.2 = +1.7 \text{ kN-m}$$

$$M_{FCB} = +1.5 + 3.2 = +4.7 \text{ kN-m}$$

Span CD sinks by 20 mm

$$M_{FCD} = \frac{-Wab^2}{L^2} + \frac{6EI\delta}{L^2}$$

$$= -\frac{9 \times 1 \times 2^2}{3^2} + \frac{6 \times 2 \times 10^5 \times 2.4 \times 10^6 \times 20 \times 1}{(3000)^2}$$

$$= -4 + 6.4 = +2.4 \text{ kN-m}$$

$$M_{FDC} = \frac{Wba^2}{L^2} + 6.4$$

$$= \frac{9 \times 2 \times 1^2}{3^2} + 6.4 = +8.4 \text{ kN-m}$$

Joint B

$$K_{BA} = \frac{3EI}{3} = EI \quad \Sigma k = \frac{7EI}{3} \quad D.F_{BA} = \frac{3}{7}$$

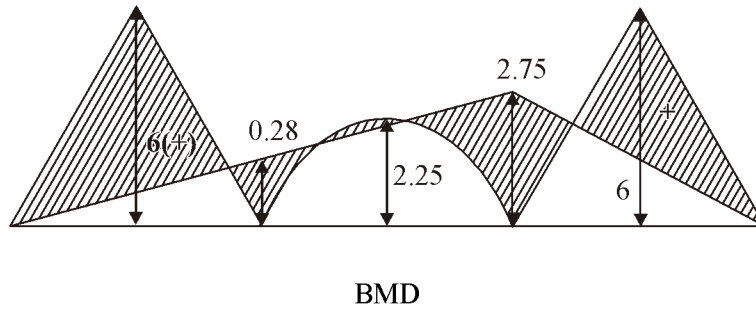
$$K_{BC} = \frac{4EI}{3} = \frac{4EI}{3} \quad D.F_{BC} = \frac{4}{7}$$

Joint C

$$K_{CB} = \frac{4EI}{3} \quad \Sigma k = \frac{7EI}{3} \quad D.F_{CB} = \frac{4}{7}$$

$$K_{CD} = \frac{3EI}{3} = EI \quad D.F_{CD} = \frac{3}{7}$$

A	B		C		D	
	3/7	4/7	4/7	3/7		
-9.4	-3.4	+1.7	+4.7	+2.4	+8.4	FEM
+9.4	+4.7			-4.2	-8.4	Balance A and D and CO to B and C.
0	+1.3	+1.7	+4.7	-1.8	0	Initial Moments
	-1.29	-1.71	-1.66	-1.24		Balance
	+0.36	-0.83	-0.85			Carry over
		+0.47	+0.49	+0.36		Balance
		+0.25	+0.24			Carry over
	-0.11	-0.14	-0.14	-0.10		Balance
		-0.05	-0.07			Carry over
	+0.02	-0.03	+0.04	+0.03		Balance
0.00	+0.28	-0.28	+2.75	-2.75	0.00	Final Moments



◀ Solution End.

6.(c) Write a short note on the tensile strength test of mild steel with the stress-strain curve.

तनाव विकृति वक्र के साथ मृदु इस्पात के तनन सामर्थ्य परीक्षण पर एक संक्षिप्त टिप्पणी लिखिए।

[10 Marks]

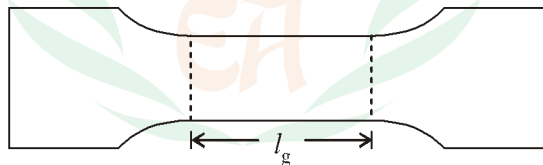
Sol.:

Tensile strength test of mild steel :

This test is of static type i.e. load is increased comparatively slowly from zero to a certain value.

Standard specimens are used for the tensile test. There are two types of specimens used for this purpose.

- (i) Circular cross-section.
- (ii) Square/Rectangular cross section



l_g = gauge length

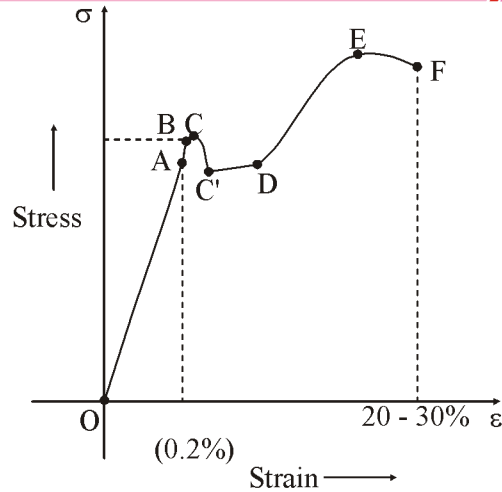
The gauge length to diameter/size ratio is generally kept as 4 : 1.

This test is carried out on Tensile testing machine and the following steps are performed to conduct this test.

- (i) The end of specimens are secured in the grips of the testing machine.
- (ii) There is a unit of applying a load to the specimen with a hydraulic or mechanically drive.
- (iii) There must be a some recording device by which you should be able to measure the final output in the form of load or stress. So the testing machines are often equipped with the pendulum type lever pressure gauge and hydraulic capsule and the stress v/s.

Strain diagram is plotted which has the following shape.

The stress-strain curve for mild-steel has been shown below :



In the above diagram:

- OA → limit of proportionality
- linear Elastic Range
- OB → Elasticity limit
- AB → Non Linear Elastic Range
- C → Upper Yield Point
- C' → Lower yield point
- (Due to slip of carbon atoms)
- CD → Yield-Plateau
- DE → Strain hardening Range
- E → Ultimate Stress
- F → Failure point / Rupture Point
- EF → Necking Zone.

ENGINEERS ACADEMY

